SIMONS

Electrical Constants of a

500 Mile Transmission Line

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ELECTRICAL CONSTANTS OF A 500 MILE TRANSMISSION LINE

BY

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B. S. University of Illinois, 1912

THESIS

Submitted in Partial Fulfillment of the Requirements for the

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Alexander McDougall Simons

ENTITLED Electrical Constants of a 500 Mile Transmission Line.

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Science in Electrical Engineering

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INTRODUCTION.

With an ever decreasing coal supply and increasing demand for power, it is becoming more and more evident that the world must utilize to the utmost all other sources of energy. The most available of these are the rivers, streams, and other bodies of water. There are millions of horse power going to waste in these places that could be harnessed and made to do the work of man. This has been done at such places as Keokuk, Iowa, where power is generated for St. Louis; Niagara Falls, New York; Cedejolo, Italy, where power is generated for Milan; Kykkelsrud, Norway; and so forth. At these places engineers have shown that it is possible to convert the energy, which heretofore has been allowed to dissipate itself, into a form useful to humanity. However, as a rule, the places where it is possible to construct such plants are often far away from the centers of industry as well as being usually handicapped by transportation difficulties, etc. Thus the proposition of transmitting the energy to where it can be used becomes of great importance. There is no form of energy more suitable for transportation purposes than that of electricity.

The limiting condition to this method of transporting energy is distance. It is a comparatively easy matter now to design a transmission line of from 50 to 100 miles in length; but when one takes into consideration such propositions as that of transporting the abundant energy of the Victoria Falls, South America, to where there is a demand for it as at the Rand Mines, some 700 miles away,

other losses appear than those which occur in short lines. This is due to the fact that in order to keep the cost of the conductors within a reasonable range, a very high voltage must be used. At large voltages, the electricity tends to leak from the wires into the surrounding air. This phenomena, known as corona, is accompanied by large power losses, and can be seen by the glowing of the transmission wires on dark nights. The cost of building lines for such high voltages is very great as steel towers must be used instead of the ordinary poles, and the insulators supporting the wires become very large and, therefore, very expensive. Of course in addition to the corona loss there are the losses always found in a transmission line, as those due to resistance, inductance, and capacity. All of these losses increase with the distance the energy must be transmitted, so everything tends to lower the efficiency of a very long line. At present the largest system in use in the United States is the one operated by the Southern Power Company between Great Falls, South Carolina and Durham, North Carolina. Some 40,000 kilowatts are being distributed from this 200 mile line at a voltage of 100,000 volts. However, at present a line is under construction which will, when completed, carry some 120,000 kilowatts from the mountains to Sacremento and San Francisco. This line will be 275 miles long, and will be operated at 150,000 volts. With such large voltages impressed upon the line, if the conductors are small in diameter, there will be large corona losses; but if the diameter of the wires is large enough this loss will disappear. However, when the conductors are very large, in size, they are also very expensive, a condition which must be avoided. This can be

done by using spaced cables instead of single wires for the conductors. Such cables are built of several small wires separated by a core so that the diameter of the cable will be several times that of a single wire containing the same cross section area of conducting material. There are two materials generally used for conductors, Aluminum and Copper. Both of these give good results, though they have different constants. Copper costs less per pound and has a lower resistance, while aluminum gives a larger surface area and therefore has less corona losses. The price of aluminum has, for the last ten years, been kept at such a price that it would cost about the same to build a transmission out of aluminum as to build one out of copper.

There may never be a demand for a longer transmission line in Europe or the United States than the one now under construction in California, but this is not the case in Africa or South America. As was mentioned before, engineers have been contemplating some method of utilizing the power of Victoria Falls at the Rand Mines. These mines could use about 150,000 kilowatts (200,000 horse power). As this power would have to be transmitted over 700 miles, one can see the immensity of the proposition. Of course it would be necessary under such conditions to send the power over two parallel circuits, so that even if one of these should be interrupted, the generating station need not be shut down but could send a large proportion of the normal load over the sècond circuit. As the most economical way to transmit electrical energy, known to-day, is by means of a three phase system, each one of these parallel circuits would be three phase.

A proposition can easily be conceived, of where 200,000 kilowatts are needed at some location, 500 miles from any place where that amount of energy could be developed. In order to transmit so large an amount of power such a long distance, the highest possible voltage that can be handled should be used. With the present development of insulators, about 200,000 volts between the line and grounded neutral is the maximum voltage that can be handled. This corresponds to 300,000 volts between lines for a three phase system. In order not to exceed 200,000 volts between line and grounded neutral at the generator, a voltage of 150,000 volts to neutral at the receiving end of the line has been assumed. In this supposed case, as engineering practice demands two separate circuits, 100,000 kilowatts would have to be transmitted over each circuit, or in case of a three phase system 33,300 kilowatts per phase. Consequently each conductor of the circuits would have to be designed to carry 33,300 kilowatts, which would mean that 225 amperes per conductor must be delivered at the receiving end.

CHAPTER I.

The Design of the Conductors for the Line.

The first thing to do in order to find the constants of a transmission line is to determine the size of wire necessary to transmit the power economically. There are many formulae for determining the cross sectional area of an economical wire, but one of the newest and probably best is the following one, developed by Mr. M. Takahashi in his thesis for a Master's degree at the University of Illinois, 1913.

$$A = \frac{P}{E \cos \theta} \sqrt{\frac{K\ r\ X}{3000\ C\ W\ h}} \qquad (1)$$

In this formula,

A = area in circular mils .

P = power per conductor in kilowatts at full load.

E = voltage between lines in kilovolts at receiving end of the line.

$\cos \theta$ = power factor at receiving end of the line.

K = market price in dollars per kilowatt per year at the receiving end of the line.

X = load factor in percent.

C = cost in dollars of conductor per pound.

W = weight of conductor per circular mil mile in lbs.

r = specific resistance of material for one mile of wire one circular mil in cross section area.

h = depreciation of conductor plus interest on investment in percent.

The principal way in which this formula differs from almost all others is that the length of the line does not appear directly.

This is due to the fact that it is taken into account in choosing the voltage of the line; and, therefore, is not necessary to consider it separately. In considering the system mentioned in the introduction, there would be 100,000 kilowatts per circuit and the voltage between lines would be 260,000 volts. Therefore, in determining the size of wire necessary the following values for the quantities in equation (1) would have to be taken into consideration.

The power delivered per phase (P) would be 33,300 kilowatts.

The voltage between lines at the receiving end of the line (E) would be 260,000 volts.

The power factor at the load (Cos θ) would depend upon the conditions, but under ordinary conditions will be between 0.80 lag and Unity, as most of the machines in use draw a lagging cunent.

The cost of power at the receiving end of the line (K) will vary considerably with the conditions under which the power is developed. It will be somewhat higher in the case that the electricity is generated by means of steam than if by means of water power. A good working average is $50 per kilowatt year, which value has been used in the following calculations.

The load factor (X) is the ratio of the energy actually delivered to that which would be delivered provided the generating plant was run continually at full load. After studying conditions in various places it will be seen that 40 percent. is a fair value.

The cost of the conductors (C) is $0.15 per pound for copper and $0.30 per pound for aluminum. These values are taken not from the present price of wire but from an average of the prices over a number of years.

The weight of the conductor (W) is 0.016 pounds per circular

mil mile for copper and 0.005 pounds per circular mil mile for aluminum.

The specific resistance (r) varies with the temperature. At 25° C it will be 56,000 ohms for one mile of wire one circular mil in cross section area, if the wire is made of copper, or 90,000 ohms if the wire is made of aluminum.

The depreciation and interest (h) has been taken as 10 percent. This allows 5 percent. for interest and 5 percent. for depreciation.

The following calculations have been made for both copper and aluminum wire, with the conditions where Cos θ varies from 0.8 lag to unity by intervals of 0.05. From these values the curves shown in Plate No. I has been ploted, showing the relation between the cross-section of wire needed and the power factor.

Table No. I.

Part 1.

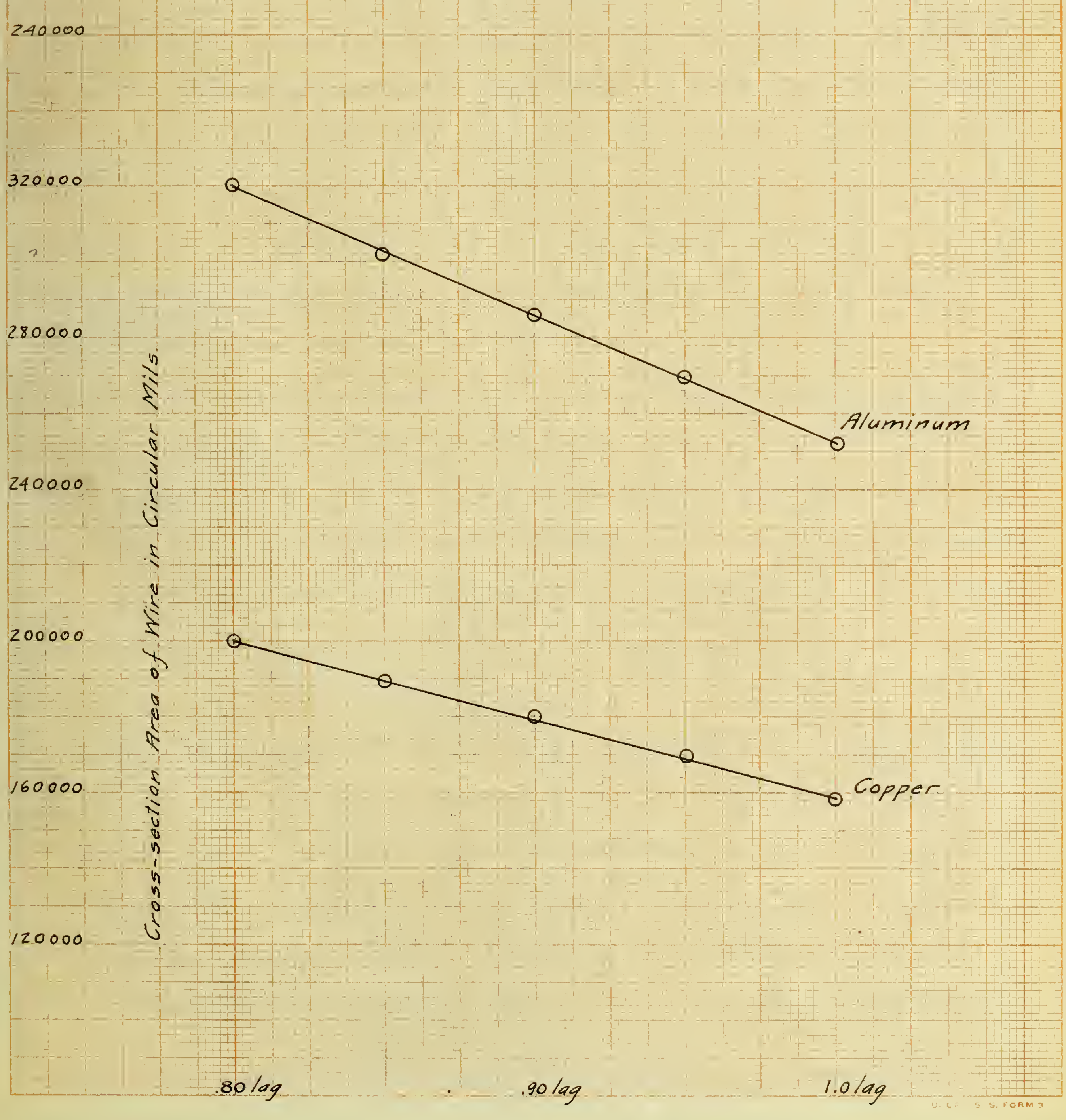
Material	Copper				
P	33,300 kilowatts				
K	50 dollars				
r	56,000 ohms				
X	40 percent.				
C	0.15 dollars				
W	0.016 pounds				
h	10 percent.				
Cos θ	.80 lag	.85 lag	.90 lag	.95 lag	Unity
E Cos θ	208	221	234	247	260
K r X	112,000,000	112,000,000	112,000,000	112,000,000	112,000,000
$\sqrt{\frac{K\ r\ X}{5000\ C\ W\ h}}$	1250	1250	1250	1250	1250
$\frac{P}{E\ \text{Cos}\ \theta}$	160	151	143	135	126
A (Circular mils)	200,000	189,000	179,000	169,000	158,000

Table No. I

Part 2.

Material	Aluminum.				
P	33,300 kilowatts				
K	50 dollars				
r	90,000 ohms				
X	40 percent.				
C	0.30 dollars				
W	0.005 pounds				
h	10 percent.				
K r X	180,000,000				
$\sqrt{\frac{K\ r\ X}{3000\ C\ W\ h}}$	2000				
Cos θ	.80 lag	.85 lag	.90 lag	.95 lag	Unity
E Cos θ	208	221	234	247	260
$\frac{P}{E\ \cos\theta}$	160	151	143	135	126
A (circular mils)	320,000	302,000	286,000	270,000	252,000

Plate No. 1
Relation between Power Factor
and Cross-section Area of Wire.



From Table No. I and Plate No. I, it will be seen that it is necessary to use No. 0000 Brown and Sharpe gauge copper wires or aluminum wires 0.6 inches in diameter. The best way to build the line is in two parallel circuits. These circuits may both be suspended from the same towers or else two entirely different sets of towers. Of course the first method is the cheaper and probably would be used even though it makes it more dangerous for the repair man working on one circuit while the other is in use. In either case the conductors should be suspended 60 feet from the ground and 10 feet from each other. This is about the minimum spacing which will give good results with the high potential to be used on this line. If the conductors were hung higher up or farther apart, the increase in the cost of the towers would be prohibitive, while not changing the constants of the line to a great extent. As this line is to operate at a very high voltage, there may be corona losses, especially as the wires have a small radius of curvature. Such losses are due to the leakage of current from the conductors, and can be seen on dark nights as then the wires glow brilliantly. This glow begins at a voltage called the visual critical voltage, and increases rapidly as the voltage increases beyond that point. This luminosity of the air surrounding the conductors is accompanied by a considerable loss of power. This loss of energy should begin, theoretically, at the visual critical voltage; however, some losses actually take place at lower voltages due to roughness of the conductors, caused by weathering, the collection of dirt on the wires, imperfections in the manufacture of the wires, etc. The voltage at which these losses begin is called the disruptive

critical voltage. The larger the wires the higher the disruptive critical voltage and, consequently, the less the corona losses. Thus one would at first glance decide to use the aluminum wire. However, it will be shown that these losses are very great even with the aluminum wire, so no advantage would be gained by using it. Therefore, the calculations for the line have been made for copper conductors.

Extended investigations on transmission lines, by F. W. Peek, Jr., have shown that the corona loss in fair weather follows the following law:

$$P = 1.61 \ a f (e - e_0)^2 \quad - - - - - - - - \quad (2)$$

where P = power loss in kilowatts per mile length of line.

e = effective voltage to neutral in kilovolts.

e_0 = disruptive critical voltage to neutral.

f = frequency.

and where a is given by the equation

$$a = \frac{K}{\delta} \sqrt{\frac{r}{S}} \quad - - - - - - - - - - \quad (3)$$

when K is a constant (.00344, see page).

δ = density of the air referred to standard conditions (76 cm. pressure and 25° C.)

r = radius of conductor in centimeters.

and S = distance between wires.

This equation does not hold for $e < e_0$ as then there can not be any corona losses.

The empirical equation as found by F. W. Peek, Jr., for the value of e_0 is

$$e_0 = M_o g_o \ \delta r \log \frac{S}{r} \text{ (in Kilovolts)} \quad - - - \quad (4)$$

where g_0 is the disruptive gradient of air in kilovolts per centimeter at standard atmospheric conditions. This value is 29.8

Kilovolts (Maximum) per centimeter or 21.1 Kilovolts (effective, assuming sine wave conditions) per centimeter, and is constant for all sizes of wire and for all frequencies.

M_0 is a constant depending upon the surface conditions of the conductor. In the case of weather beaten wires this constant will be as low as 0.93, while in the case of smoothly polished wires it will be as high as unity. For the calculations of this formula see page .

By replacing in equation (2) the value of "a" as found by equation (3) and that of "lo" given by equation (4), the total loss (P) is given by the equation

$$P = \frac{1.61\ K\ l}{\delta} \sqrt{\frac{r}{S}}\ f \left[e - M_0 g_0\ \delta r \log \frac{S}{r} \right]^2 \quad \text{--(5)}$$

In this equation

l = length of line in miles.

$K = 344 \times 10^{-5}$

g_0 = 21.1 kilovolts per centimeter (effective)

$\delta = \dfrac{3.92\ b}{273 + t}$ (b = barometer pressure in centimeters. (t = temperature in degrees centigrade.

M_0 varies from .93 to 1.00.

When b = 76 centimeters and t = 25 degrees centigrade, the value of δ is one.

From equation (5) it is apparent that the corona losses vary directly as $\sqrt{r}$, directly as f, inversely as $\sqrt{S}$, inversely as δ and directly as the square of the difference between the voltage to neutral and the disruptive critical voltage. Experiments have shown that the humidity of the atmosphere does not affect the corona,

neither do heavy winds; while fogs, rain, snow, and sleet lower the descriptive critical voltage, and, consequently, increase the losses. The difference between the disruptive and visual critical voltages varies somewhat with the size of the conductor, being larger for a small wire than for a large one. The following equation for the visual critical voltage (e_v) has been developed by F. W. Peek, Jr and was published in Volume XXX of the "Transactions of the American Institute of Electrical Engineers".

$$e_v = M_v g_o \; r \quad \delta \left(1 + \frac{0.301}{\sqrt{r}}\right) \log \frac{S}{r} \text{ - - - - - - } (6)$$

where e_v = the visual critical voltage in kilovolts and

$M_v = M_o$ and varies from 0.93 to 1.0 depending upon the roughness of the surface of the conductor. If g_o is expressed in effective kilovolts, e_v will be in effective kilovolts, while if g_o is expressed in maximum units, e_v will be in maximum units also.

From equations (4) and (6) it is possible to calculate the disruptive and visual voltages, while from equation (5), the corona loss may be found. Substituting for K and 1 equation (5) becomes

$$P = 1.61 \text{ X } 344 \text{ X } 10^{-5} \text{ X } 500 \sqrt{\frac{r}{S}} \; f \left\{ e - \frac{M_o \, g_o \; r}{\log \frac{S}{r}} \right\}^2 \qquad (5')$$

The value of δ may be taken as unity, that of M_o as 0.95, and that of e as 150 kilovolts (effective), and that of g_o as 21.1 kilovolts (effective). The radius of the wire necessary to carry the current has been found to be .23 inches or .585 centimeters if copper is used, or .3 inches or .76 centimeters if aluminum is used. Also S = 10 feet or 304.8 centimeters.

And $\log \frac{S}{r} = 6.2558$ for the copper wire and 5.994 for the aluminum wire.

Therefore $e_0 = 73.5$ kilovolts for the copper wire and 91.5 kilovolts for the aluminum wire.

$a\sqrt{\frac{r}{S}} = .0438$ for the copper wire and .0548 for the aluminum wire.

The value of f may vary, but American engineers use either one of the two standard frequencies (25 or 60 cycles per second). As P varies directly with f, the loss will be smaller at 25 cycles than at 60. Taking $f = 25$ cycles, P = 17,700 kilowatts per wire for the copper conductor, or 13,000 kilowatts per wire for the aluminum. The total loss for each of the two parallel circuits would be 53,100 kilowatts or 39,000 kilowatts depending upon the material of the conductors. As these losses are over one-third of the total power delivered, something must be done to lessen them. This can be done by varying some of the factors, as r, S, or f. The smaller the corona loss, the more economical will be the operation of the line; therefore, the line should preferably be designed so that there would be no corona losses. In order to have this condition P, in equation (2), must equal zero. This is possible only when e equals or is less than e_0, as a and f can never equal zero. The value of e_0 from equation (4) is $M_0 g_0 \delta r \log \frac{S}{r}$, therefore for the maximum value of e for no corona loss would be when

$$e = M_0 g_0 \delta r \log \frac{S}{r} \qquad \text{- - - - - - - - - - - -(7)}$$

In equation (7) the only variables are S and r and as S is already nearly as large as it is practical to make it, r must be varied until the conditions are fulfilled. However, if r is increased

very much, the conductors will soon become very expensive, so some method of reducing the cost of the conductors must be found. This can be done by using a spaced cable whose cross section is something like that shown in Fig. 1. The radius of the cable (OP) may be taken as the value of r in the preceeding equations. The number of strands in the cable will depend entirely upon the voltage impressed. In order to get the size of the cable which will have no corona losses, equation (7) must be solved for r. This solution gives equation (8).

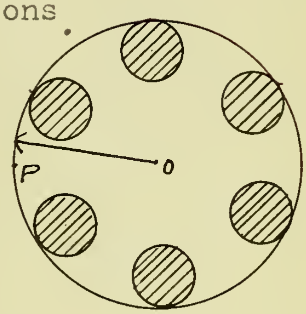


Fig. 1.

$$r \log\frac{s}{r} = \frac{e}{M_o g_o} \quad \text{- - - - - - - - - - - - - -} \quad (8)$$

It would req uire a very complicated method to get this equation in the form of $r = F(e,s)$, so some other way to solve it must be found. Such a method would be to assume values of r and calculate the value of the left hand side of the equation, and than plott a curve between these values and r. As the value of the right hand side of the equation is constant for any value of e, the value of r to satify equation (8) may be found from this curve. This solution is worked out in Table No. 2 and Plate No 2.

Table No. 2

r (Centimeters)	$\frac{S}{r}$	$\log \frac{S}{r}$	$r \log \frac{S}{r}$
.5	610	6.41	3.21
1.0	305	5.72	5.72
1.5	203	5.31	7.99
2.0	152	5.03	10.05
2.5	122	4.80	12.01
3.0	102	4.64	13.90

If e is taken as 150 kilovolts, g_0 as 21.1 kilovolts per centimeter (effective), δ as 1, and m_0 as .93, then from equation (8) $r \log \frac{S}{r} = 7.64$, and r would be 1.4 centimeters. If, however, e is taken as 200 kilovolts, $r \log \frac{S}{r}$ would equal 10.20 and r would be 2.05 centimeters.

Plate No. 2.
Relation between r and r log $\frac{s}{r}$

As the conductors must be so large it would be too expensive to use solid wires, so built up cables of several strands must be used. As this would neutralize the advantage of large surface area that the aluminum wire has over the copper, the cables would probably be built up of copper wires. As the conductors of each circuit must contain 200,000 circular mils of cross sectional area each; if there are three wires in each cable, each wire must have a cross sectional area of 67,000 circular mils. This would mean that No. 2 Brown and Sharpe gauge would be used in building the cable. If there are six wires in the cable, each wire would have to have a cross sectional area of only 33,000 circular mils and would be built up of No. 5 Brown and Sharpe gauge wire. These cables (of 3 or 6 strands) are designed under the supposition that the surface of the cable will be the surface from which the corona loss flows. If the number of wires is small, this may not be the case and the disruptive gradient of the air may be exceeded. This matter will be investigated in the next chapter, first for the cable built up of three wires and then for the one built up of six.

CHAPTER II.

Testing of the Cables Designed in Chapter I for Corona Loss.

In order to prove that there is no corona loss from the cables designed in Chapter I, it will be necessary to prove that a potential gradient of 21.1 kilovolts per centimeter (effective) has not been exceeded at any part of the cable. If g_0 represents the potential gradient, d E the voltage generated in moving a unit charge thru a distance dx in an electrostatic field set up by the charge Q, R the force set up by the electrostatic field, Φ the flux in the field, and A the area of the field; then

$$R = \frac{4\pi Q}{K A}, \quad \text{and}$$

$$d E = R \, dx$$

$$G_0 = -\frac{d E}{d x} = -R = \frac{-4\pi Q}{K A} \qquad (9)$$

where K is a constant depending upon the material in which the electrostatic field is set up. In a transmission line, the material surrounding the conductors is air, and as for air the value of the constant, K, is unity, K may be neglected in equation (9) and

$$G_0 = \frac{-4\pi Q}{A} \qquad (10)$$

In order to determine g_0 from equation (10), it will be necessary to eliminate both of the quantities, Q and A from this equation, getting them in terms which can be evaluated.

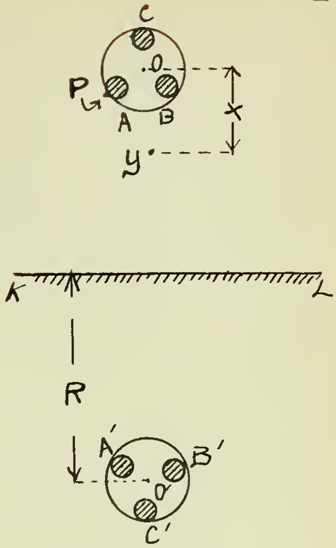


Fig. 2.

In Fig. 2, O represents one of the three cables of one of the two circuits, and O', the image of the cable below the ground line K L. A, B, and C represent the three wires in the cable and A', B', and C' represent the images of these wires respectively. Let R be the distance from the center of the cable to the ground, then OO' will be 2R. Let the radius of the wires in the conductor be r and the distance from the center of the conductor to the center of each wire be a. The wires are uniformly distributed in the cable, that is, their centers are at the vertices of an equilateral triangle, the length of whose sides is $\sqrt{3}\,a$ and the center of the triangle being O. The circumferences of the wires are tangent to the inner surface of the cable. Let Q be the charge on the conductor due to its potential above the ground, then $\frac{Q}{3}$ will be the potential of each wire, as the distance a is negligible in comparison to R. If P is at the point of tangency of wire A with the cable, then

$$\left.\begin{aligned}
E_p \text{ due to } A\,A' &= \frac{2Q}{3} \log \frac{A_2}{r_1} \\
E_p \;\text{"}\;\; \text{"}\;\; A\,B' &= \frac{2Q}{3} \log \frac{A\,B'}{r_1} \\
E_p \;\text{"}\;\; \text{"}\;\; A\,C' &= \frac{2Q}{3} \log \frac{A\,C'}{r_1} \\
E_p \;\text{"}\;\; \text{"}\;\; B\,A' &= \frac{2Q}{3} \log \frac{A\,A'}{A\,B} \\
E_p \;\text{"}\;\; \text{"}\;\; B\,B' &= \frac{2Q}{3} \log \frac{A\,B'}{A\,B} \\
E_p \;\text{"}\;\; \text{"}\;\; B\,C' &= \frac{2Q}{3} \log \frac{A\,C'}{A\,B}
\end{aligned}\right\} \;\text{-----}\; (11)$$

$$\left.\begin{aligned} E_p \text{ due to } C\,A' &= \frac{2\,Q}{3} \log \frac{A\,A'}{AC} \\ E_p \quad " \quad " \quad C\,B' &= \frac{2\,Q}{3} \log \frac{A\,B'}{A\,C} \\ E_p \quad " \quad " \quad C\,C' &= \frac{2\,Q}{3} \log \frac{A\,C'}{A\,C} \end{aligned}\right\} \quad (11)$$

and E_p is the sumation of equations (11).

But $r_2 = A\,A' = A\,B' = A\,C' = 2\,(R - \frac{a}{2}) = 2\,R - a,$

$A\,B = A\,C = \sqrt{3}a,$ and $r_1 = r$

Therefore

$$E_p = \frac{2\,Q}{3}\left[3 \log \frac{(2\,R - a)}{r} + 6 \log \frac{(2\,R - a)}{\sqrt{3}a}\right]$$

$$= 2\,Q \left[\log \frac{2\,R - a}{r} + 2 \log \frac{2\,R - a}{\sqrt{3}a}\right]$$

$$= 2\,Q \log \frac{(2\,R - a)^3}{3\,a^2\,r} \quad - - - - - - (12)$$

As E_p = the voltage of the conductors (E), solving equation (12) for Q,

$$Q = \frac{E}{2 \log \frac{(2R - a)^3}{3a^2\,r}} \quad - - - - - - - (13)$$

The voltage at any point y (see Fig. 2) may be found by the same method as that for the point P. This method will give

$$Ey = 2\,Q \left[\log \frac{A'y}{A\,y} + \log \frac{B'y}{B\,y} + \log \frac{C'y}{C\,y}\right] \quad - - -(14)$$

Now if y is taken at a distance X from point O on the line O O' below the conductor (Fig. 2) so that the distance X is small in comparison to R, then

$A'\,y = B'\,y = C'\,y = 2\,R - X$ and

$$A\,y = C\,y = \sqrt{(X - \frac{a}{2})^2 + (\sqrt{3}\,a)^2} = \sqrt{X^2 + a^2 - a\,X}$$

Also $B\,y = X + a$

And equation (14) becomes

$$E\,y = 2\,\rho \log \frac{(2\,R - X)^3}{X^3 + a^3} \quad - - - - - - (15)$$

When the points y and P are at the same potential, that is on the same equipotential surface, E_p must equal E_y, and from equations (12 and (15),

$$\log \frac{(2\,R - a)^3}{3\,a^2\,r} = \log \frac{(2\,R - X)^3}{X^3 + a^3} \quad - - - - (16)$$

which is the same as,

$$\frac{(2\,R - a)^3}{3\,a^2\,r} = \frac{(2\,R - X)^3}{X^3 + a^3} \quad - - - - - -(17)$$

When 200 kilovolts between line and neutral are impressed upon the circuits, the values of R, a, and r are 1440 inches, 0.677 inches, and .129 inches, respectively. Then X is the only variable in equation(17) and solving for X equation (18) is obtained as follows:

$$\frac{(1440 - X)^3}{X^3 + (.667)^3} = \frac{(1440 - .677)^3}{3\ (.677)^2\ .129} = 16 \text{ X } 10^9$$

or $\quad 16 \times 10^9 X^3 + 1.44 \times 10^3 X^2 + 2.003 \times 10^6 X = 1.79 \times 10^9 \qquad (18)$

From this equation it is evident that the value of X must be less than unity, and the values of the second and third terms. being so small may be neglected. Therefore, equation (18) becomes

$$16 \times 10^9\ X^3 = -1.79 \times 10^9$$

or

$$X^3 = -.112$$

and

$$X = -.482 \text{ inches}$$

The result indicates that the line O O' cuts the equipotential surface of value is E above the point O. Consequently, this surface is not a continuous, but is divided into three separate parts, each part winding around one of the wires as shown in Fig. 3 by curves A, A', and A". The equation of these surfaces, as shown by Alexander Russel in his "Treatise on the Theory of Alternating Currents", is

$$X^6 - 2a^3X^3 \cos 3\theta + a^6 = r_1^2\, r_2^2\, r_3^2 \qquad \text{- - - - - - (19)}$$

where X is the distance, O Y from the center of the cable to a point on the equipotential surfaces; a, the distance from the center of the cable to the axis of one of the conductors, as O C; θ the angle between O Y and O C; and r_1, r_2, and r_3 the distances from any point on the equipotential surface to the axes of the three wires. This equation becomes when the surface has a potential E

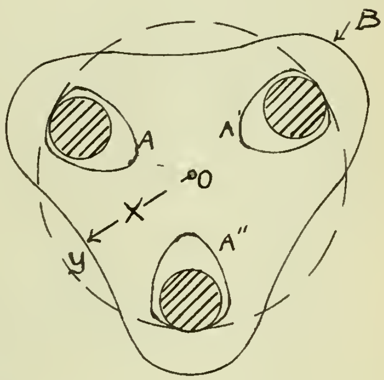


Fig. 3.

A-represents the equipotential surfaces of value E.

$$X^6 - 2\,a^3X^3 \cos 3\theta + a^6 = r^2\left(\sqrt{3}\,\{r + a\}\right)^4 \qquad \text{- - -(20)}$$

where r is the radius of the wires. Expanding and substituting the values of r and a in equation (20)

$$X^6 - .626\,X^3 \cos 3\theta + .033 = 0 \qquad \text{- - - - - - - - (21)}$$

When Y is on the line O O' below the cable, the angle θ is 60 degrees and cos 3 θ is -1, and equation (21) becomes

$$y^6 + 626\,y^3 + .033 = 0$$

Solving this for y

$$y^3 = \frac{-.628 \pm \sqrt{.385 - .136}}{2} = -.513 \text{ or } .114$$

$$y = -.805 \text{ inches or } -.485 \text{ inches.}$$

The point -.805 is at the point of tengency of the upper wire and the cable, and the point -.485 is on the line 0 0' between the point 0 and that wire. This indicates that the equipotential surface surrounds only one of the three wires, the same conclusion which was drawn from equation (16). In fact, the distance to the point of intersection of the equipotential surface and the line 0 0' as found by equation (16) is only .003 inches away from that found by means of equation (19). This variation is accounted for by the fact that several approximations were made in determining both parts of equation (16) which are not taken into consideration in equation (19), also equation (19) is for the case that the conductors are eliptical instead of circular in shape.

The equipotential surface, as shown by equations (16) and (19), tangent to the three wires is three curves. As the distance of these equipotential surfaces are taken further and further away from point 0 they become larger and larger until at the place where the product is $r_1^2\ r_2^2\ r_3^2$ becomes equal to a^6, the three curves meet at point 0 and the curve is the one called by mathematicians, the three leafed rose. After this, as the surfaces are taken further away they take the shape of curve B in Fig. 3. When the equipotential surfaces are tangent to the wires, their length will be smallest and consequently the potential gradient will be greatest at that place. If this surface had a uniform rate of curvature the potential gradient (g_0) would be the same at all places, but as can be seen from equation (19), this is not the case. Therefore, g_0 is not uniform around the wires, and the area of the equipotential surface can not be substituted in equation (10) in order to find

the value of g_o, as that equation was developed under the supposition that the rate of curvature was uniform. Consequently, some other method of finding g_o must be used.

If point y is taken on line 0 0' above the cable instead of below it then

$$A'y = B'y = C'y = 2R + X \;;\quad Ay = Cy = Cy = \sqrt{a^2 + X^2 + aX}$$

and $\quad By = X - a.$

Then equation (15) becomes

$$Ey = \log \frac{2R - X}{X^3 - a^3} \quad - - - - - - - - - - - \quad (22)$$

Differentiating equation (22) taking in account the fact that 2 R-X is practically equal to 2 R

$$\frac{dE}{dX} = -2\,\varphi\, \frac{3X^2}{X^3 - a^3} \quad - - - - - - - - - - - - \quad (23)$$

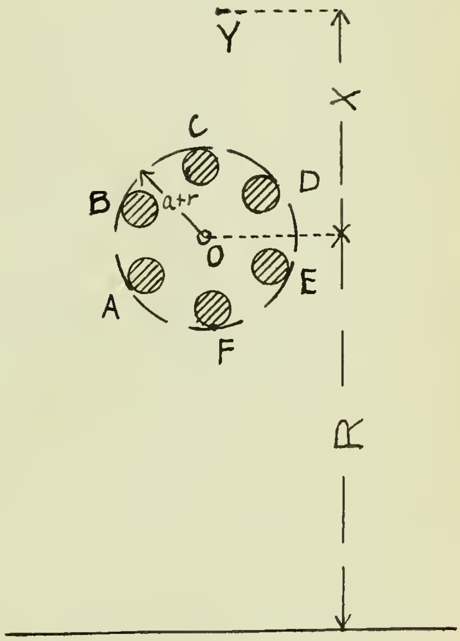
From equation (13), $\varphi = \dfrac{E}{2 \log \dfrac{(2R - a)^3}{3a^2 r}}$, therefore, as $go = -\dfrac{dE}{dX}$

$$g_o = + \frac{E}{\log \dfrac{2R - a}{3a^2 r}} \cdot \frac{3X^2}{X^3 - a^3} \quad - - - - (24)$$

As the corona losses will occur from the outside surfaces of the cables, they must accur at or near the places where the wires are tangent to the surface of the conductors. At these places X is 2.05 centimeters, a is .677 inches or 1.72 centimeters, r is .128 inches and R is 1440 inches. By substituting these values in equation (24), when E_o equals 200 kilovolts (effective), the value of g_o has been found to be 30.8 kilovolts per centimeter (effective).

But the limiting value of g_o is 21.1, so there will be corona loss from the conductors if they are built up of three wires each.

If the cable is built up of six wires, the value of O and g_o will be changed. If the wires are arranged at the vertices of a regular hexagon whose center is the center of the cable as O in Fig. 4, and if O' is the image of O, then the charge on each wire will be 1/6 of the total charge, and the voltage at the surface of one of the wires is the sum of the following thirty six expressions.



$$E_p \text{ due to } A\,A' = \frac{2\,Q}{6} \log \frac{A\,A'}{r}$$

$$E_p \text{ " " } A\,B' = \frac{2\,Q}{6} \log \frac{A\,B'}{r}$$

$$E_p \text{ " " } A\,C' = \frac{2\,Q}{6} \log \frac{A\,C'}{r}$$

$$E_p \text{ " " } A\,D' = \frac{2\,Q}{6} \log \frac{A\,D'}{r}$$

$$E_p \text{ " " } A\,E' = \frac{2\,Q}{6} \log \frac{A\,E'}{r}$$

$$E_p \text{ " " } A\,F' = \frac{2\,Q}{6} \log \frac{A\,F'}{r}$$

$$E_p \text{ " " } B\,A' = \frac{2\,Q}{6} \log \frac{B\,A'}{B\,A} \quad \text{) Six equations as above}$$

- - - - - - - - - - - - - - -

$$E_p \text{ due to } C\,A' = \frac{2\,Q}{6} \log \frac{C\,A'}{C\,A} \quad \text{) Six equations as above}$$

- - - - - - - - - - - - - - -

$$E_p \text{ due to } D\,A' = \frac{2\,Q}{6} \log \frac{D\,A'}{D\,A} \quad \text{) Six equations as above}$$

- - - - - - - - - - - - - - -

$$E_p \text{ due to } E\,A' = \frac{2\,Q}{6} \log \frac{E\,A'}{E\,A} \quad \text{) Six equations as above}$$

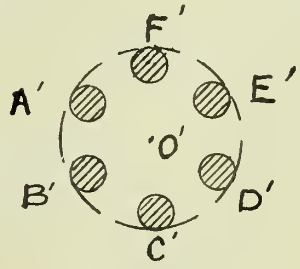


Fig. 4.

E_p due to F A' $= \frac{2\,\rho}{6} \log \frac{F\,A'}{F\,A}$) Six equations as above.

But as R, the distance from 0 to the ground, is very large in comparison to a, the distance from 0 to the axes of the wires, A A' = B B' = C C' = A B' = A C' etc. = 0 0' = 2 R - a. Also A B = A F = a, A C = A E = $\sqrt{3}$ a, and A D = 2 a, then by substituting these values in and by adding up the thirty-six expressions,

$$E_p = 2\,\rho \left[\log \frac{2\,R}{r} + 2 \log \frac{2R}{a} + 2 \log \frac{2\,R}{\sqrt{3}\,a} + \log \frac{2\,R}{2\,a}\right]$$

$$E_p = 2\,\rho \log \frac{(2\,R)^6}{6\,a^5\,r} \qquad (25)$$

$$\text{and} \quad \rho = \frac{E}{2 \log \frac{(2\,R)^6}{6\,a^5\,r}} \qquad (26)$$

Now if y is some point on the line 0 0' above the cable, and if the same reasoning is used as in deriving equation (14)

$$E_y = 2\,\rho \log \frac{(0'Y)^6}{\overline{A\,Y} \cdot \overline{B\,Y} \cdot \overline{C\,Y} \cdot \overline{D\,Y} \cdot \overline{E\,Y} \cdot \overline{F\,Y}}$$

But 0' Y = 2 R + X, A Y = E Y = $\sqrt{X^2 + a^2 + a\,X}$, F Y = X + a, B Y = D Y = $\sqrt{X^2 + a^2 - a\,X}$, and C Y = X - a. Therefore

$$\overline{A\,Y} \cdot \overline{B\,Y} \cdot \overline{C\,Y} \cdot \overline{D\,Y} \cdot \overline{E\,Y} \cdot \overline{F\,Y} = X^6 - a^6$$

$$\text{and } E_y = 2\,\rho \log \frac{(2\,R - X)^6}{X^6 - a^6} \qquad (27)$$

As X, when Y is near the cable is very small in comparison to R, 2 R - X may be taken as simply 2 R. Differentiating equation (27),

$$\frac{d\,E}{d\,X} = -2\,\rho \frac{6\,X^5}{X^6 - a^6} \qquad (28)$$

As $g_0 = - \frac{dE}{dX}$ and as ρ is given by equation (26)

$$g_0 = \frac{E}{\log \frac{(2R)^6}{6\,a^5\,r}} \cdot \frac{6\,X^5}{X^6 - a^6}$$

When the cable is built up of six wires R is 1440 inches, a is .714 inches, or 1.82 centimeters, and r is .091 inches. If the point Y is taken at the surface of one of the conductors, X is 2.05 centimeters, the radius of the conductors. If E is taken as 200 kilovolts effective, the value of g_0 will be 17.3 kilovolts (effective) per centimeter. This value is somewhat smaller than the descriptive gradient, so consequently there would be no corona loss with the conductors made up as shown in Fig. 4 (6 wires in the cable).

From these calculations it is evident that a built up cable of three wires would not do for the line while one of six would. Therefore, each conductor of the system would be made of six No. 5 Brown and Sharpe gauge copper wire. These wires are held apart by metal discs, spaced about twenty feet apart. These discs would cost about 10 cents and could be put on the wires for about 20 cents a piece, making the total cost per mile of each cable about $90.00 more than the cost of the wire. This value is very small when taken into consideration with what the additional cost would be if the conductors were made of solid copper.

CHAPTER III.

Calculation of the Constants of the Line.

Now that the size and design of the conductors has been decided upon, there is enough data known in order to calculate the constants of the line.

In calculating a short transmission line, the capacity has very little effect upon the regulation of the line, and may be neglected entirely or else assumed to be at the center of the line for purpose of calculation. These assumptions cannot be made when the line is very long, but the resistance and the inductive and capacity reactances must be taken as distributed over the entire length of line. Dr. E. J. Berg has worked out equations for the solution of lines with distributed inductive and capacity reactances. These equations are rather simple and are developed in the following manner.

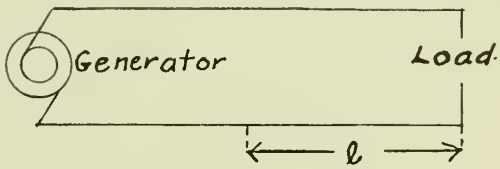


Fig. 5

In Fig. 5, E_1 and I, are the voltages and the current, respectively, at the receiving end of the line, and E_0 and I_0 are voltage and the current, respectively, at the sending end of the line. Z_0 is the vector sum of the resistance and inductive reactance and Y_0 is the factor by which E_0 should be multiplied to give the leakage current.

At a distance l from the receiving end of the line the voltage consumed is

$$d\,E = I\,Z_0\,dl \qquad (30)$$

and the leakage current is

$$d\,I = Y_o\,E\,dl \quad - - - - - - - - (31)$$

Differentiating equation (30)

$$\frac{d^2 E}{d\,l^2} = Z_o \frac{d\,I}{d\,l} = Z_o\,Y_o\,E \quad - - - - (32)$$

The solution of equation (32) is

$$E = A_1\,E^{\sqrt{Y_o\,Z_o}\,l} + A_2\,E^{-\sqrt{Y_o\,Z_o}\,l} \quad - - (33)$$

Differentiating equation (33)

$$\frac{d\,E}{d\,l} = A_1\sqrt{Y_o\,Z_o}\;E^{\sqrt{Y_o\,Z_o}\,l} - A_2\sqrt{Y_o\,Z_o}\;E^{-\sqrt{Y_o\,Z_o}\,l} \quad (34)$$

From equation (30) $I = \frac{1}{Z_o}\frac{dE}{dl}$

$$\therefore\; I = \sqrt{\frac{Y_o}{Z_o}}\,\left(A_1\,E^{\sqrt{Y_o Z_o}\,l} - A_2\,E^{-\sqrt{Y_o\,Z_o}\,l}\right) \quad - - - - (35)$$

For the entire line $l = 1$, then

$$E_o = A_1\,E^{\sqrt{Y_o\,Z_o}} + A_2\,E^{-\sqrt{Y_o\,Z_o}} \quad - - - - - (36)$$

and
$$I_o = \sqrt{\frac{Y_o}{Z_o}}\,\left(A_1\,E^{\sqrt{Y_o\,Z_o}} - A_2\,E^{-\sqrt{Y_o\,Z_o}}\right) \quad - - - - (37)$$

But
$$E^{\pm\sqrt{Y_o Z_o}} = 1 \pm \sqrt{Y_o\,Z_o} + \frac{Y_o Z_o}{2} \pm \frac{Y_o Z_o\sqrt{Y_o\,Z_o}}{2 \times 3} + - - -$$

This series is a rapidly convergent one and consequently all terms after the third may be neglected without causing much error. Then equations (36) and (37) become after substituting this series for $E^{\sqrt{Y_o\,Z_o}}$ and $E^{-\sqrt{Y_o\,Z_o}}$,

$$E_o = (A_1 + A_2)\left(1 + \frac{Y_o\,Z_o}{2}\right) + (A_1 - A_2)\sqrt{Y_o\,Z_o} \quad - - - (38)$$

$$I_o = \left[(A_1 + A_2)\sqrt{Y_o\,Z_o} + (A_1 - A_2)\left(1 + \frac{Y_o\,Z_o}{2}\right)\right]\sqrt{\frac{Y_o}{Z_o}} \quad (39)$$

As l the length of the line measured from the receiving end, must be zero at that end, and as $\dot{E} = \dot{E}_1$, and $\dot{I} = \dot{I}_1$, at that place, then from equations (33) and (35)

$$\dot{E}_1 = A_1 + A_2 \qquad (40)$$

and $$I_1\sqrt{\frac{Z_o}{Y_o}} = A_1 - A_2 \qquad (41)$$

Substituting these values in equations (38) and (39)

$$\dot{E}_o = \dot{E}_1\left(1+\frac{Y_o Z_o}{2}\right) + \dot{I}_1 Z_o \qquad (42)$$

$$\dot{I}_o = \dot{E}_1 Y_o + I_1\left(1 + \frac{Y_o Z_o}{2}\right) \qquad (43)$$

These last two equations give the values of the current and voltage in terms of the constants of the line, and the load currents and voltage. Before they can be solved these values must be found. As $Z_o = r_o - jX_o$ and $Y_o = g_o - jb_o$, the values of M_o, X_o, g_o, and b_o must be known. r_o is the total resistance of the entire line; X_o, the reactance, g_o the conductance, and b_o the susceptance of the entire line. As this is a three phase transmission line, 500 miles long, the values of X_o, r_o, g_o, and b_o will be taken for 500 miles of the cable. The value of r_o can be found from tables published in such books as the "Standard Handbook for Electrical Engineers". $X_o = 2\pi f L_o$ where f is the frequency and L_o is the total inductance of the line. L_o can be found from equation (44).

$$L = .14 \log_{10}\frac{D - r}{r} + .01524^{*} \qquad (44).$$

In this formula L is expressed in mil henrys per 1000 feet, so $L_o = 500 \times 5.28 \times L$. $g_o = 0$ as there is to be no corona loss, and as $g_o = 0$, b_o will equal $\frac{1}{X_c}$. As $X_c = \frac{1}{2\pi f C_o}$, $b_o = 2\pi f C_o$.

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*Standard handbook for Electrical Engineers, Page 43.

C (the capacity of the line) is given by the equation*

$$C = \frac{0.0073}{\log_{10}\frac{D-r}{r}} \qquad - - - - - - - - (45)$$

where r and D represent, as in equation (44), the outside radius of the cables and the distance between two of them, respectively. In equation (45), C is given in microfarads per 1000 feet, so $C_0 = 500 \times 5.28 \times C$.

In Chapter I two cables were designed, one if 150 kilovolts were impressed upon the circuit, and the other if 200 kilovolts were impressed upon it. Unless there is a negative regulation the larger cable must be used. From a test calculation it was found that the regulation would probably vary from about 25% at a power factor of 0.80 lag to about one half of that value at unity power factor. As this will mean that from 170 to 200 kilovolts between line and neutral must be impressed upon the line, the smaller cable designed in Chapter I will have corona losses. Therefore, the calculations which follow were made with the larger cable. These calculations show, as did the test calculations, that this cable is necessary and that the smaller one would not have been adequate.

The value of r_0 was found to be 137 ohms, L_0 was found to be .826 henries and C_0 was found to be 8.97×10^{-6} farads. At various frequencies, X_0 and b_0 have different values. These values have been computed and put in a table (Table No. 3)

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* Standard Handbook for Electrical Engineers, Page 46.

Table No. 3

f	g (mhos)	X (ohms)
15	.000845	78
25	.00141	130
40	.00226	208
50	.00282	260
60	.00338	312

In order to know anything about the best conditions for the operation of the line, the regulation, efficiency, line losses, and the power factor at the generator must be known for various load power factors and frequencies. Calculations were made for the power factor of the load varying from 0.75 lag to 0.75 lead for frequencies of 15, 25, 40, and 60 cycles, under full load conditions. and for the same frequencies at no load. These results are tabulated in Tables 4 to 8 inclusive. These calculations were made from equations (42) and (43) in the following manner, taking E_1 as the refference vector and using the time rotation as used by Dr. C. P. Steinmetz.

The following data is known.

E_1 = 150 kilovolts to neutral

P = 100,000 kilowatts per circuit or 33,000 kilowatts per line.

r = 137 ohms.

As soon as the frequency and P.F. have been chosen, b, X, and Cos θ, are known, Cos θ being the power factor at the load. As equation (42) is $E_0 = E_1 \left(1 + \frac{Y_0 Z_0}{2}\right) + I_1 Z_0$ the following calculations must be made in determining E_0

$$I = \frac{i_1}{P\ F} \quad \text{and} \quad i_1 = I^2 - i^2$$

$$\frac{Y_o\ Z_o}{2} = \frac{(r_o - jX_o)(-jbo)}{2} = -\frac{jb_o r_o}{2} + \frac{X_o\ b_o}{2} = -c - jb.$$

$$\text{where } c = \frac{b_o r_o}{2} \text{ and } b = \frac{X_o b_o}{2}$$

$$1 + \frac{Y_o Z_o}{2} = 1 + (-c - jb) = 1 - c - jb = a - jb$$

where $a = 1 - c$.

$$E_1 \frac{(1 + YZ_o)}{2} = e_1\ (a - jb) = e_1\ a - je_1 b = f - jg$$

where $f = e_1 a$ and $g = e_1 b$

$$I_1\ Z_o = (i_1 + ji_1')(r - jX) = i_1 r + i_1' X + j\ (i_1' r - i_1 X)$$

$$= h + jk$$

where $h = i_1 r + i_1' X$, and $k = i_1' r + i_1 X$

∴ Therefore $E_o = (f - jg) + h + jk = f + h + j\ (k - g) = e_o + je_o'$

where $e_o = F + h$ and $e_o' = k - g$

As equation (43) is $I_o = I_1\ (1 + \frac{Y_o\ Z_o)}{2} + E_1 Y_o$

the following calculations must be made in order to get I_o, P_o, $P_o - P$, the efficiency of the line, and the regulation.

$$I_1\ (1 + \frac{Y_o Z_o}{2}) = (i_1 + ji_1')(a - jb) = ai_1 + i_1' b + j(ai_1' - i_1 b)$$
$$= l + jm$$

where $l = ai_1 + i_1' b$ and $m = ai_1' - i_1 b$

$E_1 Y_0 = (e_1)(-j b_0) = -j e_1 b_0 = -j\, n$, where $n = e_1 b_0$

Therefore $I_0 = 1 + j\,m - j\,n = 1 + j\,(m - n) = i_0 + j\,i'_0$

where $i_0 = 1$ and $i'_0 = m - n$

P_0 (Power given to each line by generator) $= e_0\, i_0 + e'_0\, i'_0$

Loss $= P_0 - P$

$$\text{Efficiency of line} = \frac{P_0 - P}{P}$$

$$\text{Power factor at generator} = \frac{P_0}{E_0\, I_0} = \frac{P_0}{\sqrt{(e_0)^2 + (e'_0)^2}\sqrt{(i_0)^2 + (i'_0)^2}}$$

$$\text{Regulation} = \frac{E_0 - E}{E}$$

A numerical sample of these calculations is as follows.

If P.F. $= .75$ lag, and $f = 15$, then

$E_0 = 150{,}000$ volts $= 150$ kilovolts $\qquad Y_0 = -.000845\, j$

$i = 222$ amperes $\qquad Z_0 = 137 - 78j$

$I = \dfrac{222}{.75} = 296$ amperes

$i'_1 = \sqrt{\overline{296}^2 - \overline{222}^2} = 195$ amperes.

$\dfrac{Y_0\, Z_0}{2} = \dfrac{(-.000845\, j)\,(137 - 78\, j)}{2} = -.0329 - .058\, j = (c - j b)$

$1 + \dfrac{Y_0\, Z_0}{2} = 1 + (-.0329 - .058\, j) = .968 - 116\, j = (a - j b)$

$E_1 \left(1 + \dfrac{Y_0\, Z_0}{2}\right) = 150\,(.968 - .058\, j) = 145.8 - 8.7\, j$ kilovolts.

$= (f - j\, g)$

$I_1 Z_0 = (i_1 + j\, i'_1)\,(r - j\, X) = (222 + 195j)(137 - 78j)$

$= 30.2 + 15.2 + j(26.7 - 17.3) = 45.4 + 11.4j = (h + jk)$

$E_0 = I_1 Z_0 + E_1 \left(1 + \dfrac{Y_0\, Z_0}{2}\right) = (h + j\, k) + (f - j\, g)$

$= 45.4 + 11.4j + 145.8 - 8.7j = 191 + 2.7j = (e_0 + e_0')$

$= 191.1$ kilovolts.

$E_1 Y_0 = 150\ (-.000845\ j) \times 1000 = -122\ j\ = (-j\ n)$

$$I_1 \left(1 + \frac{Y_0 Z_0}{2}\right) = (i_1 + j\ i_1')(a - j\ b) = (222+195j)(.968 - .058j)$$

$$= 216 + 1.3 + j(189.1 - 12.9) = 227.1 + 176.2j$$

$$= (l + j\ m)$$

$$I_0 = (l + j\ m) + (-jn) = 227.1 + 176.2j - 122j = 227 + 54.2j = i_0 + ji_0' = 236 \text{ amperes.}$$

$$P_0 = e_0\ i_0 + e_0'\ i_0' = 227 \times 191 + 2.7 \times 54.2 = 43400 + 150$$

$$= 43550 \text{ kilowatts}$$

$$\text{Loss} = P_0 - P = 43550 - 33{,}300 = 10250 \text{ kilowatts}$$

$$\text{Efficiency} = \frac{P}{P_0} \times 100 = \frac{33300}{43550} \times 100 = 76.4 \text{ percent.}$$

$$E_0\ I_0 = 191.1 \times 236 = 45200 \text{ kilovolt amperes.}$$

$$\text{Power factor at generator} = \frac{P_0}{E_0 I_0} = \frac{43550}{45200} = .965 \text{ lag}$$

$$\text{Regulation in percent.} = \frac{E_0 - E}{E} \times 100 = \frac{191.1 - 150}{150} = 27.4\ \%$$

In order to be able to compare the results of these calculations with one another, they have been plotted in three sets of curves.* The first set (Plate No.3) shows the relation between the efficiency of the line and the power factor of the load for the four different frequencies; the second set (Plate No. 4) shows the relation between the regulation (in percent.) and the power factor of the load for the different frequencies; and the third set (Plate No. 5) shows the relation between the power factor of the generating end of the line to that of the receiving end. These curves are all taken for full load conditions.

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*For these calculations see pp. 38 - 46 and for the curves see pp. 47-49

TABLE NO. 4

Calculations of the transmission line under full load conditions when f = 15 cycles.

P_1 = 33000 kilowatts E_1 = 150 kilovolts Z_o = 137 - 78j ohms Y_o = - .00845j mhos.

$1 + \frac{Y_o Z_o}{2}$ = a - jb = 968 - .058j E_1 (a - jb) = 145.8 - 8.7 j kilovolts $E_1 Y_o$ = - 122j amp.

P. F.	lag .75	lag .80	lag .85	lag .90	lag .95	unity	lead .95	lead .90	lead .85	lead .80	lead .75	
i_1	222	222	222	222	222	222	222	222	222	222	222	amperes
i_1'	195	164	136	107	72	0	-72	-107	-136	-164	-195	"
I_1	296	278	261	247	233	222	233	247	261	278	296	"
$i_1 r$	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	kilovolts
$i_1' x$	15.2	12.8	10.6	8.4	5.6	0	- 5.6	- 8.4	-10.6	-12.8	-15.2	"
$i_1 r + i_1' x$	45.4	43	40.8	38.5	35.8	30.2	24.6	21.9	9.6	17.3	15.	"
$i_1' r$	26.7	22.5	18.6	14.6	9.9	0	-9.9	-14.6	-18.6	-22.5	-26.7	"
$i_1 x$	-17.3	-17.3	-17.3	-17.3	-17.3	-17.3	-17.3	-17.3	-17.3	-17.3	-17.3	"
$i_1' r + i_1 x$	11.4	5.2	1.3	- 2.7	- 7.4	-17.3	-27.2	-31.9	-30.9	-39.8	-44	"
e_o	191	188.8	186.8	184.3	181.5	176	170.4	167.7	160.4	163.1	160	"
e_o'	2.7	- 3.5	- 7.4	-11.4	-16.1	-26	-35.9	-40.6	-43.6	-48.5	-52.7	"
E_o	191.1	188.9	187	184.6	182	178	174	172.5	171.3	170	169	"
$a i_1$	216	216	216	216	216	216	216	216	216	216	216	amperes
$i_1' b$	11.3	9.5	7.9	6.2	4.2	0	- 4.2	- 6.2	- 7.9	- 9.5	-11.3	"

TABLE NO. 4 (Cont.)

Calculations of the transmission line under full load conditions when f = 15 cycles

i_o	227.3	225.5	223.9	222.2	220.2	216	211.8	209.8	208.1	206.5	204.7	Amp.
ai_1'	189.	159.1	132.1	104	70	0	-70	-104	-132.1	-159.1	189	
bi_1	-12.9	-12.9	-12.9	-12.9	-12.9	-12.9	-12.9	-12.9	- 12.9	-12.9	-12.9	"
i_o'	54.2	26.2	- 2.8	-30.8	-64.9	-134.9	-202.9	-239	-267	-294	-314	"
I_o	236	227	224	225	232	252	292	317	338	359	372	"
$e_o\ i_o$	43400	42600	41800	41000	40000	38000	36100	35200	34400	33500	32700	K.W.
$e_o'\ i_o'$	-150	-90	20	350	1040	3490	7270	9660	11700	14300	16500	"
P_o	42250	42570	41820	41350	41040	41490	43370	44860	46100	47800	49400	"
Loss	9920	9180	8490	8020	7710	8160	10040	11530	12770	14470	16070	"
Efficiency	77	73.8	79.6	80.4	81	80.2	75.8	74.5	72.3	69.6	67.5	%
$E_o\ I_o$	45200	43200	41900	41600	42200	44800	50700	54700	58000	61000	63000	KVA
Generator P.F.	.97*	.985*	.997*	.994**	.975**	.925**	.855**	.822**	.795**	.785**	.772**	
Regu-	41.1	38.9	37	34.6	32	28	24	22.5	21.3	20	19	K.V.
lation	27.4	25.9	24.6	23	21.3	18.7	16	15	14.2	13.3	12.7	%

* Means lagging power factors.
** Leading power factors.

TABLE NO 5

Calculations of the transmission line under full load conditions when f = 25 cycles

P_1 = 33000 kilowatts E_1 = 150 kilovolts Z_o = 137 - 130 ohms Y_o = - .00141 mhos

$1 + \frac{Y_o Z_o}{2}$ = a - jb = .901 -096j E_1 (a - jb) = 135 - 14.4j kilovolts $E_1 Y_o$ = - 212j amperes.

P.F.	.75*	.80*	.85*	.90*	.95*	unity	.95**	.90**	.85**	.80**	.75**	
i_1	222	222	222	222	222	222	222	222	222	222	222	Amp.
i_1'	195	164	136	107	72	0	-72	-107	-136	-164	-195	"
I_1	296	278	261	247	233	222	233	247	261	278	296	"
i_1r	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	KV
$i_1'x$	25.4	21.4	17.7	13.9	9.35	0	-9.35	-13.9	-17.7	-21.4	-25.4	"
$i_1r+i_1'x$	55.6	51.6	47.9	44.1	40.5	30.2	20.9	16.3	12.5	8.8	4.8	"
$i_1'r$	26.7	22.5	18.6	14.6	9.4	0	-9.4	-14.6	-18.6	-22.5	-26.7	"
i_1x	-29	-29	-29	-29	-29	-29	-29	-29	-29	-29	-29	"
$i_1'r+i_1x$	-2.3	-6.5	-10.4	-14.5	-19.1	-29	-38.7	-53.6	-47.6	-51.5	-55.7	"
e_o	190.6	186.6	182.9	179.1	175.5	165.2	155.9	151.3	147.5	143.5	139.8	"
e_o'	-16.7	-20.9	-24.8	-28.9	-33.5	-43.4	-53.1	-58	-62	-65.9	-70	"
E_o	191	187	184	181	178.5	171	164	162	160	158	156.6	"
ai_1	191	191	191	191	191	191	191	191	191	191	191	Amp.
bi_1'	18.7	15.7	13.0	10.3	6.4	0	-6.4	-10.3	-13.0	-15.7	-18.7	"

TABLE NO. 5 (Continued)

Calculations of the transmission line under full load conditions
when f = 25 cycles

i_o	209.7	206.7	204	201.3	197.9	191	84.1	180.7	178	175	173.3	Amp.
ai_1'	176	148	123	96	65.4	0	-65.4	-96	-123	-148	-176	"
bi_1	-21.3	-21.3	-21.3	-21.3	-21.3	-21.3	-21.3	-21.3	-21.3	-21.3	-21.3	"
i_o	-57.3	-85.3	-110.3	-137.3	-177.9	-233.3	-298.3	-329.3	-356.3	-381.3	-409.3	"
I_o	217	226	232	244	266	301	350	374	398	422	445	"
e_oi_o	39940	38600	37300	36050	34700	31600	28700	27300	26400	25200	24100	"
$e_o'i_o'$	960	1780	2740	3980	5950	10100	15900	19100	22200	25100	28600	KW.
P_o	40900	40380	40040	40030	40650	41700	44600	46400	48600	50300	52700	"
Loss	7600	7050	6710	6700	7300	8400	11300	13100	15300	17000	19400	"
Efficiency	79	79.8	80.6	80.7	79.5	77.5	72.4	59.5	66.5	64.2	61.3	%
E_oI_o	41600	42300	42800	44100	47500	51500	57400	60500	63800	66500	69500	KVA
Generator P.F.	.982**	.995**	.938* *	.91**	.855**	.81**	.78**	.767**	.762**	.757**	.757**	
Regu-	41.5	37	34	31	28.0	21	14	12	10	8	6.5	KV
lation	27.3	24.7	22.6	20.6	19	14	9.3	8	6.7	5.3	4.3	%

* Means lagging power factor.
** Leading power factor.

TABLE NO 6

Calculations of the transmission line under full load conditions when f = 40 cycles.

P_1 = 33000 kilowatts E_1 = 150 kilovolts Z_o = 137-208j ohms Y_o = 00226 mhos

$1 + \frac{Y_o Z_o}{2} = a - jb = .769 - .152j$ $E_1 (a - jb) = 115 - 22.8j$ kilovolts $E_1 Y_o = - 33j$ amp.

P.F.	.75*	.80*	.85*	.90*	.95*	unity	.95**	.90**	.85**	.80**	.75**	
i_1	222	222	222	222	222	222	222	222	222	222	222	Amp.
i_1'	195	164	136	107	72	0	-72	-107	-136	-164	-195	"
I_1	296	278	261	247	233	222	233	247	261	278	296	"
$i_1 r$	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	KV
$i_1' x$	40.6	34.2	28.3	22.2	15	0	-15	-22.2	-28.3	-34.2	-40.6	
$i_1 r + i_1' x$	70.8	64.4	58.5	52.4	45.2	30.2	15.2	8	1.9	-4.0	-10.4	"
$i_1' r$	26.7	22.6	18.6	14.6	9.9	0	- 9.9	-14.6	-18.6	-22.5	- 26.7	
$i_1 x$	-46.2	-46.2	-46.2	-46.2	-46.2	-46.2	46.2	-46.2	-46.2	-46.2	-46.2	"
$i_1' r + i_1 x$	-19.5	-23.7	-27.6	-31.6	-36.3	-46.2	-56.1	-61.8	-64.8	-68.7	-72.9	"
e_o	185.8	179.4	173.5	167.4	160.2	145.2	130.2	123	116.9	111	104.4	"
e_o'	-42.3	-46.5	-50.4	-54.7	-59.1	-69.0	-78.9	-84.6	-87.6	-91.5	-95.7	"
E_o	190.8	185	180	175.5	170	161	152	149.5	146	144	142	"
$a i_1$	171	171	171	171	171	171	171	171	171	171	171	Amp.
$b i_1'$	29.6	25	30.7	16.3	11	0	-11	-16.3	-20.7	-25	-29.6	"

TABLE NO 6. (Continued)

Calculations of the transmission line under full load conditions when
f = 40 cycles.

i_o	200.6	196	191.7	187.3	182	171	160	154.7	150.4	146	141.4	Amp.
ai'_1	150	126	105	82	55	0	- 55	-82	-105	-126	-150	"
bi_1	-34	-34	-34	-34	-34	-34	-34	-34	-34	-34	-34	"
i'_o	-217	-241	-262	-284	-312	-367	-422	-449	-472	-493	-517	"
I_o	298	310	324	341	362	401	450	475	495	513	530	"
e_oi_o	37100	35200	33300	31400	29200	24800	20800	19000	17600	16200	14800	KW.
$e'_oe'_o$	9160	11200	13200	15500	18500	25300	33300	38000	41400	45200	49500	"
P_o	46260	46400	46500	46900	47700	50100	54100	57000	59000	61400	64200	"
Loss	12960	13100	13200	13600	14400	16800	20800	23700	25700	28100	30900	KW.
Efficiency	72.1	71.9	71.6	71.2	70	66.1	61.5	58.5	56.5	54.3	52	%
E_oI_o	56900	57500	58400	60000	61500	64500	68500	71000	72500	74000	75705	KVA.
Generator P.F.	.813**	.807**	.797**	.781**	.770**	.775**	.779**	.803**	.815**	.827**	.853**	Lead
Regulation	40.8	35	30	35.5	20	11	2.0	- .5	- 4.0	- 6.0	- 8.0	KV.
	27.3	23.4	20	17	13.3	7.3	0.7	- .33	- 2.7	- 4	- 5.3	%

* Means lagging power factors.
** Leading power factors.

TABLE NO 7

Calculations of the transmission line under full load conditions when f = 60 cycles

P_1 = 33000 kilowatts E_1 = 150 kilovolts Z_o = 137-312j ohms Y_o = - .00338 mhos

$1 + \frac{Y_o Z_o}{2}$ = a - jb = E_1 (a-jb) 71.2 - 33.9j kilovolts $E_1 Y_o$ = 507 amperes

P. F.	.75*	.80*	.85*	.90*	.95*	unity	.95**	.80**	.85**	.80**	.75**	
i_1	222	222	222	222	222	222	222	222	222	222	222	Amp.
i_1'	195	164	136	107	72	0	-72	-107	-136	-164	-195	"
I_1	296	278	261	347	233	222	.233	247	261	278	296	"
$i_1 r$	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	KW.
$e_1' x$	60.8	51.2	42.4	33.4	22.4	0	--22.4	-33.4	-42.4	-51.2	-60.8	"
$i_1 r + i_1' x$	91	81.4	72.6	63.6	52.6	30.2	7.8	- 3.2	-12.2	-21.2	-30	"
$i_1' r$	26.7	22.5	18.6	14.6	9.9	0	-9.9	-14.6	-18.6	-22.5	-26.7	"
$i_1 x$	-67.2	-67.2	-67.2	-67.2	-67.2	-67.2	-67.2	-67.2	-67.2	-67.2	-67.2	"
$i' r + i_1 x$	-40.5	-45.7	-49.4	-53.4	-57.3	-67.2	-77.1	-81.8	-85.8	-89.7	-93.9	"
e_o	162.2	152.6	143.8	134.8	123.8	101.4	79	68	59.	50.2	41.2	KV.
e_o'	-74.4	-79.6	-83.3	-87.3	-91.2	-101.1	-110	-115.7	-119.7	-123.6	-127.8	"
E_o	178.3	172.2	166	160.2	153.9	143.2	135.2	133.5	133.2	133.3	133.9	"
$a i_1$	105	105	105	105	105	105	105	105	105	105	105	Amp.
$b i_1'$	44	37	30.8	24	16.2	0	-16.2	-24	-30.8	-37	-44	"

TABLE NO 7 (Continued)

Calculations of the transmission line under full load conditions when f = 60 cycles

i_o	149	142	135.8	129	121.2	105	89.8	81	74.2	68	61	Amp.
ai_1	92.0	78	64.5	51	34	0	-34	-51	-64.5	-78	-92.5	"
bi_1	-50	-50	-50	-50	-50	-50	-50	-50	-50	-50	-50	"
i'_o	-464	-479	-493	-506	-523	-557	-591	-608	-621	-635	-649	"
I_o	487	500	512	521	536	567	598	614	625	639	652	"
$e_o i_o$	24200	21600	19500	27400	15000	10600	7100	5500	4400	3400	2500	KW
$e'_o i'_o$	34600	38100	41000	44200	47700	56400	65000	70300	74400	78500	82900	"
P_o	58800	59700	60500	61600	62700	67000	72100	75800	78800	81900	85400	"
Loss	25500	26400	27200	28300	29400	33700	38800	42500	45500	48600	52100	"
Effi- ciency	56.7	55.7	55	54	53.0	49.6	46	44.	42.2	40.7	39	
$E_o I_o$	87000	86100	85000	83600	82500	81200	80900	82200	83400	85000	87200	KVA
Generator PF.	.675**	.695**	.712**	.737**	.760**	.825**	.893**	.923**	.945**	.963**	.977	lead
Regu-	28.3	22.2	16	10.2	3.9	-6.8	-14.8	-16.5	-16.8	-16.7	-16.1	KV
lation	18.8	14.8	10.7	6.8	2.6	4.5	9.9	-11	-11.2	-11.1	-10.7	%

* Means lagging power factor.
** Leading power factors.

TABLE NO 8.

Calculations of transmission line under no head conditions with f varying from 15 to 60.

E_1 = 150 kilovolts I_1 = o r_0 = 137 ohms.

f	15	25	40	60	cycles
x_0	-78	-130	-208	-312	ohms
b_0	-.00085	-.00141	-.00242	-.00338	mhos
a	.968	.901	.769	.475	
b	.058	.096	.152	.226	
e_0	145	135	115	71.2	kilovolts
e_0'	-8.7	-14.4	-22.8	-33.9	"
E_0	146	136	117	825	"
I_0	-122	212	333	507	"

Full load generator current, P.F. = .80 lag

227	226	310	500

Full load generator current, P.F. = .85 lag

224	232	324	512

Full load generator current, P.F. = .95 lag

236	266	367	536

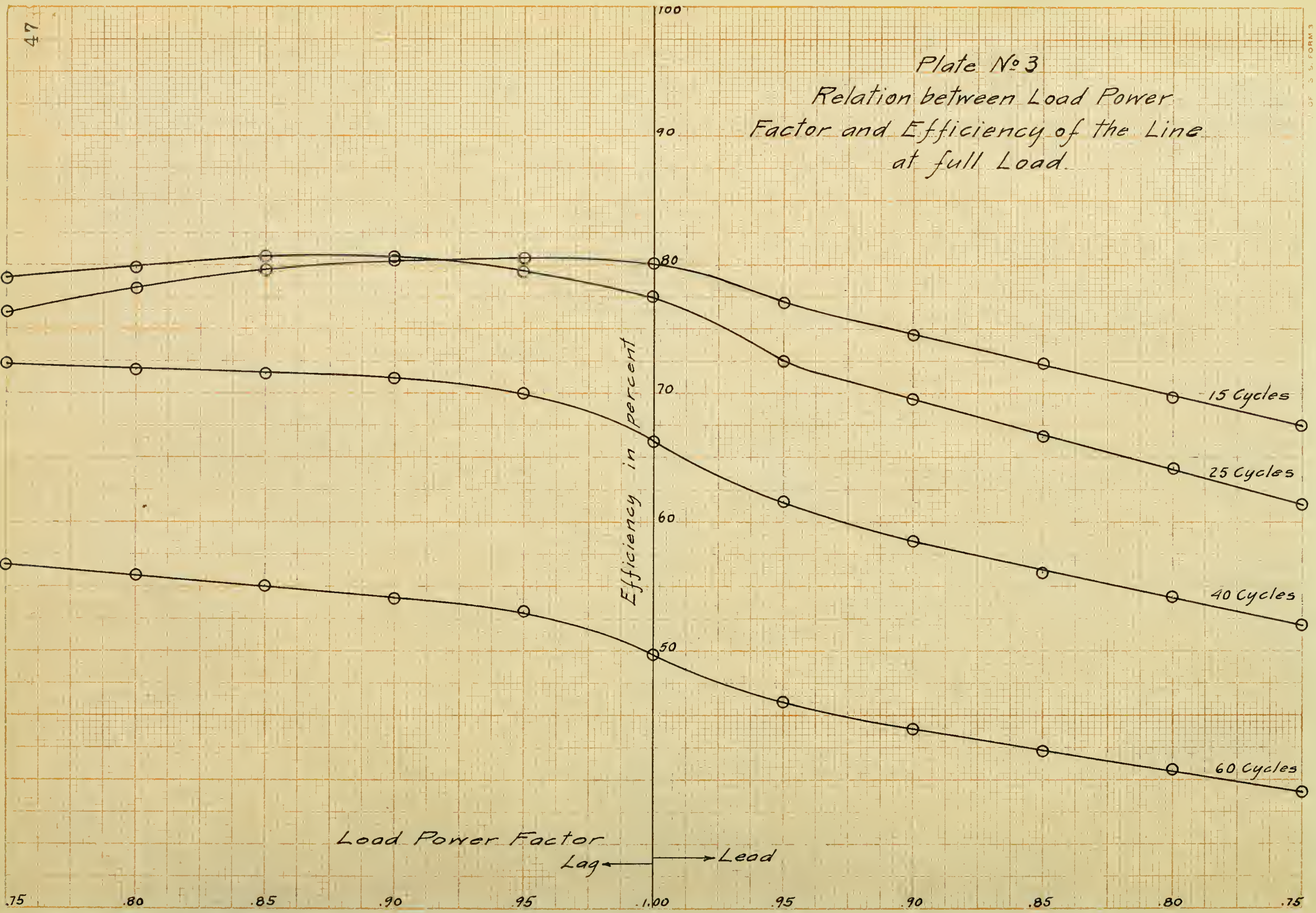
Plate No. 3
Relation between Load Power Factor and Efficiency of the Line at full Load.
Efficiency in percent
100
90
80
70
60
50
Load Power Factor
Lag
Lead
.75
.80
.85
.90
.95
1.00
.95
.90
.85
.80
.75
15 Cycles
25 Cycles
40 Cycles
60 Cycles

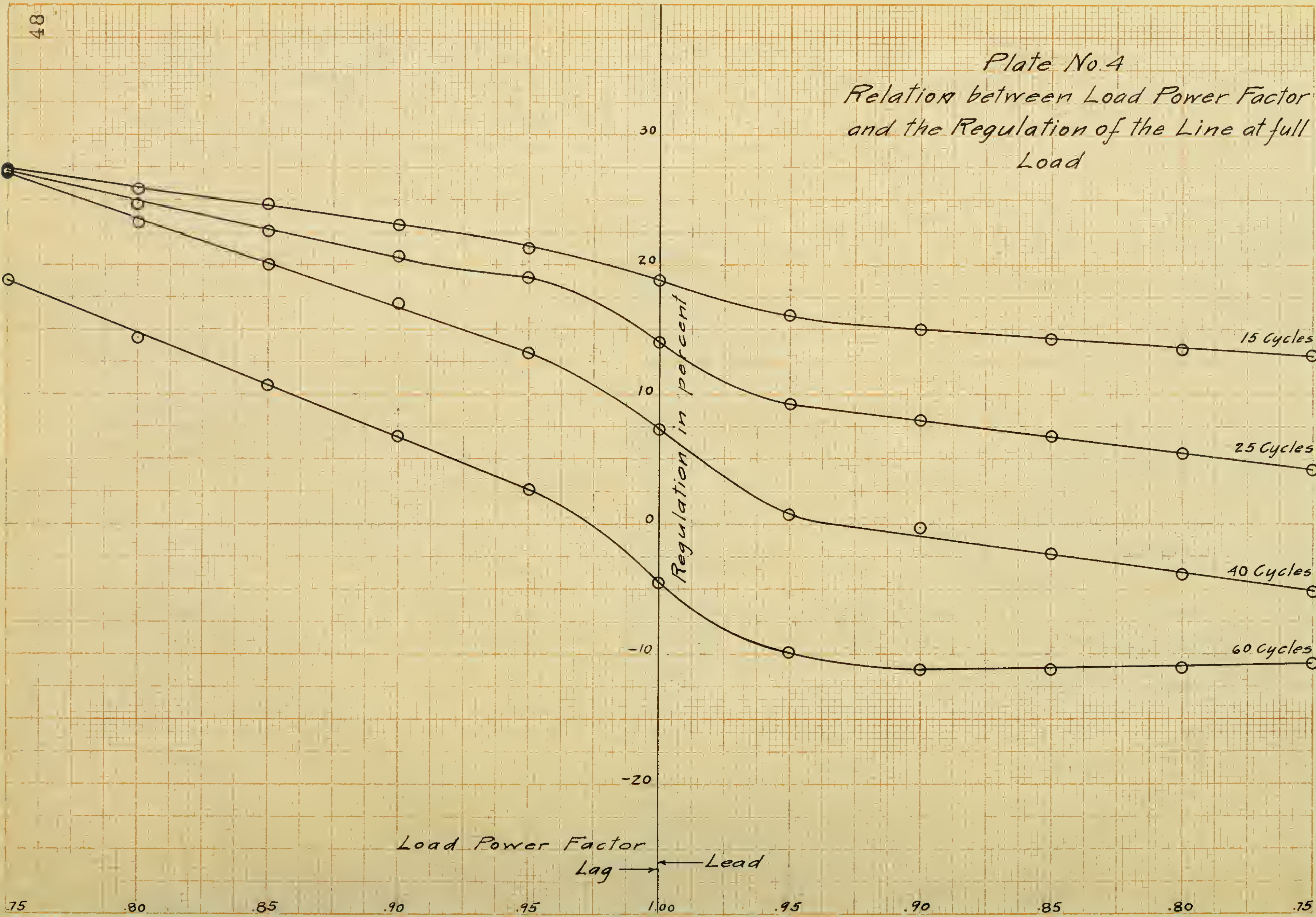

Plate No. 4
Relation between Load Power Factor and the Regulation of the Line at full Load

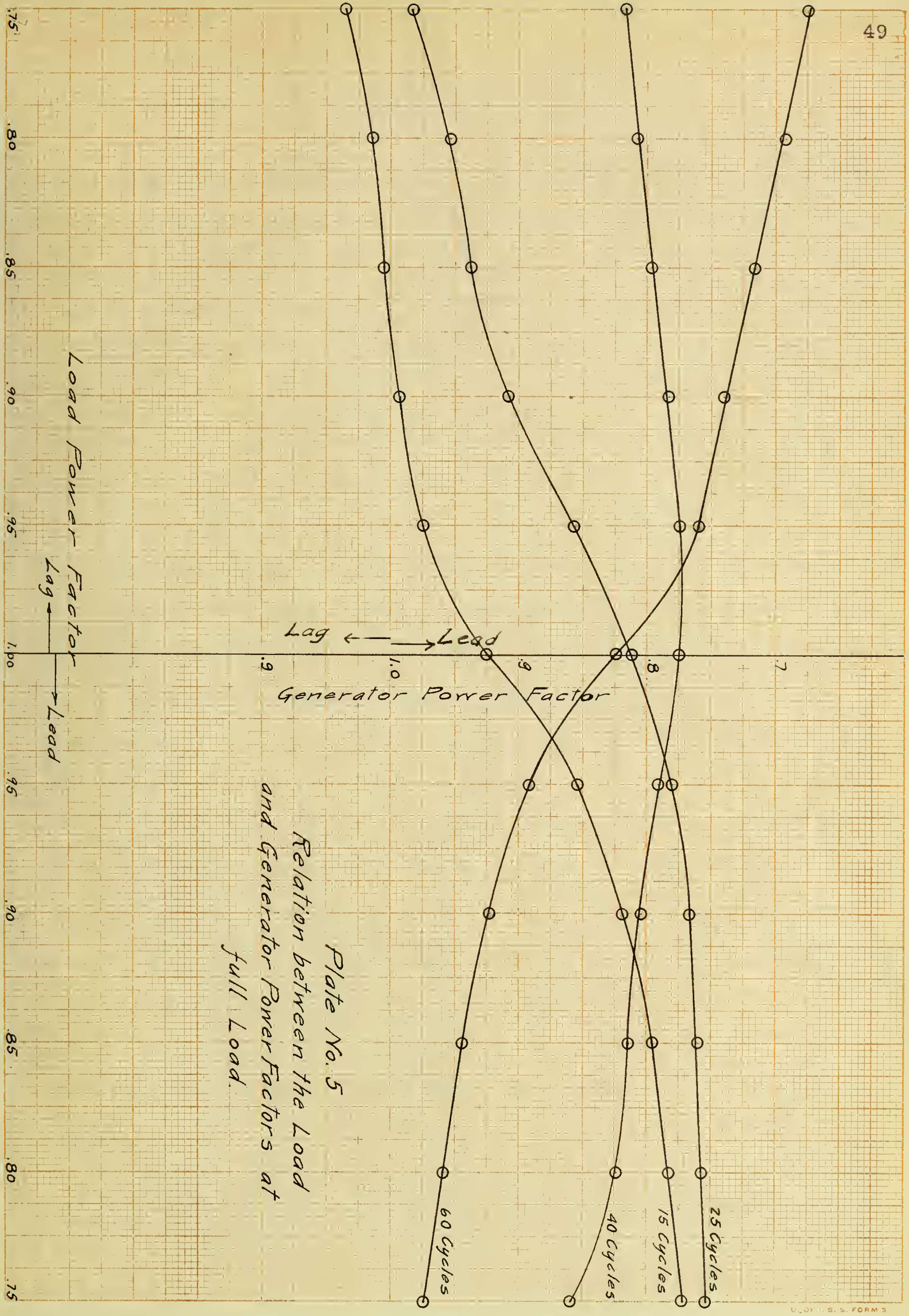

Plate No. 5
Relation between the Load
and Generator Power Factors at
full Load.

CONCLUSION.

Because of the fact that there is less possibility of any trouble putting the line out of commission, when the power is transmitted over two parallel circuits, it has proven good practice to build long high tension transmission lines in two separate circuits (each circuit three phase). These circuits are suspended from the same towers, usually one on either side of the tower. Each one of the circuits should be designed to carry one half of the power, which in this case is 200,000 kilowatts. In Chapter I, it has been shown that in order to carry this amount of power in two circuits and yet have no corona loss at the voltage used (260 kilovolts between lines at load) it will be necessary to have for each conductor a built up cable containing three or more wires. The total cross sectional area of these wires must be 200,000 circular mils, so if there are three wires in the cable each one must be a No. 2 Brown and Sharpe gauge, or if six wires per cable are used, they must be No. 5 Brown and Sharpe gauge. The diameter of these cables, if there is to be no corona loss, with 200 kilovolts impressed upon the line, must be 4.1 centimeters or 1.55 inches. In Chapter II, it was shown that there would be corona loss from the cable if it was built up of three wires, while there would not be such losses if the conductors contained six wires each, with the conductors spaced 10 feet apart and hung 60 feet from the ground.

From a study of Plates 3 to 5, and of Tables 4 to 7, inclusive, it is seen that efficiency is better for all cycles with a

lagging power factor at the load, than with a leading one. At 15 or 25 cycles, the line has a great deal larger efficiency than at the larger frequencies. The greatest efficiency (80.7) occurs with a load power factor of 0.90 lag when the line is operated at 25 cycles. When the line is operated at 15 cycles the best efficiency is 80.5 % with a load power factor of 0.95 lag. At 40 or 60 cycles the efficiency is so low (a maximum of 72 % with a current of 40 cycles and 57 % with a 60 cycle current) that it would not be practical to operate at these frequencies. The regulation would be better at 25 cycles, while the generator power factor would be better at 15 cycles. From Table 8 it is seen that if the line is operated at 25 cycles, the current would be almost constant from no load to full load. In addition, 25 cycles is a standard frequency, while 15 cycles is not.

Therefore, the conditions under which the line should be operated are that the frequency should be 25 cycles per second and the load power factor should be 0.90 lag. These conditions give an efficiency of 80.7 percent. and a regulation of 20.6 percent. while the power factor at the generator would be 0.91 lead. The constants of the line when operated under these conditions would be

Length of line - - - - - - - - - - - - - - - - 500 miles
Number of circuits - - - - - - - - - - - - - - 2 (each 3 phase)
Frequency - - - - - - - - - - - - - - - - - -25 cycles.
Power factor of load - - - - - - - - - - - - - .90 lag
" " of generator - - - - - - - - - - .91 lead
Voltage between lines at the load - - - - - - 260 kilovolts
" " " " " generator - - - - 313 "

Voltage to neutral at the load - - - - - - - - - -	150 kilovolts
" " " " " generator - - - - - - - --	181 "
Full load current per phase at the load - - - - -	247 amperes.
" " " " " " " generator - -	244 "
No " " " " " " " - -	212 "
Resistance of one conductor - - - - - - - - - - - -	137 ohms
Inductive reactance of one conductor - - - - - - -	130 "
Susceptance of one conductor - - - - - - - - - - - -	.000141 mhos.
Line regulator - - - - - - - - - - - - - - - - - - -	22.6 percent.
Power lost per phase - - - - - - - - - - - - - - -	6,700 kilo-watts.
Total power lost (both circuits) - - - - - - - - -	40,200 "
Efficiency of line - - - - - - - - - - - - - - - -	80.7 %
Distance between wires - - - - - - - - - - - - - -	10 feet.
Distance of circuits above ground - - - - - - -	60 feet.

REFERENCES.

(1) "Analysis of Modern Hydro-Electrical Developments"
by M. Takahashi. (Chapter V).

(2) "The Law of Corona and Dialectric Strength of Air"
by F. W. Peek, Jr. (Published in the "Transactions of the American Institute of Electrical Engineers" Vol. XXX, Part 3.

(3) "A Treatise on the Theory of Alternating Currents"
by Alexander Russell (Chapter V).

(4) Dr. E. J. Berg's lecture on transmission lines with distributed inductance and capacity.